Introduction to Z_n with AdditiveGroup package

```
<< AdditiveGroupMinimal`
```

```
MakeMinimalGroup[6]
```

```
Estimated time is: 0.00031104 seconds.

Computation time: 0. seconds.
```

```
Zn
```

```
{0, 1, 2, 3, 4, 5}
```

```
Subgroups[]
```

```
{{0, 1, 2, 3, 4, 5}, {0, 2, 4}, {0, 3}, {0}}
```

```
Sns
```

```
{{0, 1, 2, 3, 4, 5}, {0, 2, 4}, {0, 3}, {0}}
```

```
{3⊕4, 5⊕5, 0⊕5}(*addition*)
```

```
{1, 4, 5}
```

```
{3¯, 5¯, 0¯}(*inverser*)
```

```
{3, 1, 0}
```

```
{4⊕3¯, 2¯⊕2, 3¯⊕3¯}
```

```
{1, 0, 0}
```

```
Names["AdditiveGroupMinimal`*"]
```

```
{AdditiveGroupMinimalPackage, CayleyTable, ElementOrder, ElementOrders,
 InstanceSubgroups, MakeMinimalGroup, MakeMinimalGroupInstance, N0,
 N1, Sns, SubgroupGenerators, Subgroups, SubgroupsAndGenerator, Zn}
```

```
{N0, N1}(* gruppens ordning och antal undergrupper *)
```

```
{6, 4}
```

```
SubgroupGenerators[]
```

```
{0, 1, 2, 3}
```

```
G0 = MakeMinimalGroupInstance[12];(*G0 kontext med separat grupp*)
```

```
Estimated time is: 0.00124416 seconds.

Computation time: 0. seconds.
```

```
G0[[1]]
```

```
{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}
```

```
G0[[5]](*dess undergrupper*)
```

```
{{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11},
 {0, 2, 4, 6, 8, 10}, {0, 3, 6, 9}, {0, 4, 8}, {0, 6}, {0}}
```

```
Sns(*huvudgruppen kvar att utforska*)
```

```
{{0, 1, 2, 3, 4, 5}, {0, 2, 4}, {0, 3}, {0}}
```

```
<< AdditiveGroupBasics`
```

```
Names["AdditiveGroupBasics`*"]
```

```
{AdditiveGroupBasicsPackage, Containment, ContainmentGenerators,
 ContainmentHierarchy, ContainmentHierarchyNonTrivial, ContainmentIndexes,
 ContainmentIndexesHierarchy, ContainmentIndexesHierarchyNonTrivial,
 ContainmentIndexesNonTrivial, ContainmentMatrix, ContainmentNonTrivial,
 ContainmentSizes, GeneratorsAndOrder, HasseDiagram,
 NonUnitMinimalSubgroupGenerators, SubgroupOrders, ZeroMeetingSubgroups, Zeros}
```

```
MakeMinimalGroup[12]
```

```
Estimated time is: 0.00124416 seconds.
```

```
Computation time: 0. seconds.
```

```
SubgroupsAndGenerator[]
```

```
⟨|0 → {0}, 1 → {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11},
 2 → {0, 2, 4, 6, 8, 10}, 3 → {0, 3, 6, 9}, 4 → {0, 4, 8}, 6 → {0, 6}|⟩
```

```
SubgroupOrders[]
```

```
{1, 2, 3, 4, 6}
```

```
SubgroupGenerators[]
```

```
{0, 1, 2, 3, 4, 6}
```

```
GeneratorsAndOrder[]
```

```
⟨|0 → 1, 1 → 12, 2 → 6, 3 → 4, 4 → 3, 6 → 2|⟩
```

```
ZeroMeetingSubgroups[] // Column(*undergrupper som bara har enheten 0 gemensamt*)
```

```
{{0, 3, 6, 9}, {0, 4, 8}}
{{0, 4, 8}, {0, 6}}
```

```
Subgroups[] // Column
```

```
{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}
{0, 2, 4, 6, 8, 10}
{0, 3, 6, 9}
{0, 4, 8}
{0, 6}
{0}
```

```
Sns[[4]]
```

```
{0, 4, 8}
```

```
Containment[4] // Column (*undergrupper som innehåller den fjärde undergruppen*)
```

```
{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}
{0, 2, 4, 6, 8, 10}
{0, 4, 8}
```

```
<< AdditiveGroup`
```

MakeMinimalGroup[6]

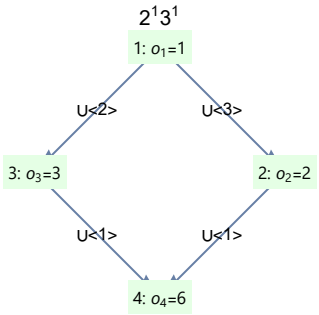
```
Estimated time is: 0.00031104 seconds.

Computation time: 0. seconds.
```

SubgroupsAndGenerator[]

$$\langle|0 \to \{0\},\ 1 \to \{0,\ 1,\ 2,\ 3,\ 4,\ 5\},\ 2 \to \{0,\ 2,\ 4\},\ 3 \to \{0,\ 3\}|\rangle$$

HasseGraphEdges[](*sambandet mellan undergrupper m.a.p. inklusion*)



G1 = AdditivePermutationGroup[];(*mathematica representation*)

{GroupOrder[G1], N0}

```
{6, 6}
```

{Sns[[3]], "+", Sns[[2]], "=", Sns[[3]] ⊕ Sns[[2]]}(* addition är polymorf till undergrupper *)

```
{{0, 3}, +, {0, 2, 4}, =, {0, 1, 2, 3, 4, 5}}
```

SubgroupProducts[] // MatrixForm(*all parvisa produkter*)

$$\begin{pmatrix} \{0,1,2,3,4,5\} & \{0,1,2,3,4,5\} & \{0,1,2,3,4,5\} & \{0,1,2,3,4,5\} \\ \{0,1,2,3,4,5\} & \{0,2,4\} & \{0,1,2,3,4,5\} & \{0,2,4\} \\ \{0,1,2,3,4,5\} & \{0,1,2,3,4,5\} & \{0,3\} & \{0,3\} \\ \{0,1,2,3,4,5\} & \{0,2,4\} & \{0,3\} & \{0\} \end{pmatrix}$$

SubgroupProducing[3] // Grid(*indikerar undergruppspar vars produkt är den tredje*)

```
0 0 0 0
0 0 0 0
0 0 1 1
0 0 1 0
```

Names["AdditiveGroup`*"]

```
{AdditiveGroupPackage, AdditivePermutationGroup, AdditivePermutationSubgroups,
 AggregatedSubgroupProducing, ContainmentAndComplement,
 ContainmentAndComplementNonTrivial, ContainmentIndexesFromGeneratorNonTrivial,
 CyclesList, CyclesMap, DivergerMatrix, DivergerMatrixBool, DivergerPairs,
 GeneratorSpan, HasseGraph, HasseGraphEdges, HasseWeb, HasseWebGraphEdges,
 InnerAut, NonEntanglingPaths, NonEntanglingPathsIndexed, PartitioningSubgroups,
 Permutation, SubgroupIntersections, SubgroupOrder, SubgroupProducing,
 SubgroupProductBoundaries, SubgroupProductBoundary, SubgroupProducts,
 SubgroupSymmetricDiffs, SubsettingPaths, SubsettingPathsContainmentIndexes,
 SubsettingPathsIndexes, vtxpnl$6219, WholeGroupProducts, ZeroMeetFactorisation}
```

```
<< AdditiveGroupQuotients`
```

```
MakeGroup[6, True]; (*andra argumentet om kvotgrupper ska genereras med gruppen*)
```

```
Estimated time is:          -6
1.40772 10 seconds.
Computation time: 0. seconds.
```

```
{Sns[[3]], Cosets[3]} (*tredje undergruppen och dess sidoklasser*)
```

```
{{0, 3}, {{0, 3}, {1, 4}, {2, 5}}}
```

```
Sns[[1]] \ Sns[[3]] (*kvotgruppsoperatorn*)
```

```
{{0, 3}, {1, 4}, {2, 5}}
```

```
QuotientGroups[][[1]] (* vanligen enbart representant för varje sidoklass*)
```

```
{{0}, {0, 1}, {0, 1, 2}, {0, 1, 2, 3, 4, 5}}
```

```
(*fullständig representation exporteras men döljs under Private`*)
AdditiveGroupQuotients`Private`QuotientGroupsFull[][[1]]
```

```
{{{0, 1, 2, 3, 4, 5}}, {{0, 2, 4}, {1, 3, 5}},
 {{0, 3}, {1, 4}, {2, 5}}, {{0}, {1}, {2}, {3}, {4}, {5}}}
```

```
Indexes[] (*antalet sidoklasser för varje kvotgrupp*)
```

```
{1, 2, 3, 6}
```

```
QuotientGroups[][[2]] (*andra undergruppens kvotgrupper*)
```

```
{{0}, {0, 1, 2}, {0, 2, 4}}
```

```
QuotientGroupGenerators[][[1]] (*icke-triviala generatorer*)
```

```
{1, 1, 1}
```

```
QuotientKernels[][[1]] (*dess kärnor under kanoniska homomorpfismen*)
```

```
{{0, 1, 2, 3, 4, 5}, {0, 2, 4}, {0, 3}, {0}}
```

```
QuotientPGroupRanks[][[1]] (*rank via mathematica PermutationGroup*)
```

```
{1, 2, 3, 6}
```

```
Names["AdditiveGroupQuotients`*"]
```

```
{AdditiveGroupQuotientsPackage, Canonical, Coset, CosetByInverse,
 CosetNumbers, Cosets, CosetSizeExtremes, CosetSizeMaxima, CosetSizeMinima,
 CosetSizes, CosetsLeft, CosetsPowerSet, Css, Extend, Extending, Index, Indexes,
 IsNormal, Kernel, MakeGroup, N2, NonTrivialQuotientKernels, QuotientGroup,
 QuotientGroupGenerators, QuotientGroupLasts, QuotientGroups, QuotientGroupSizes,
 QuotientGroupsPartition, QuotientGroupsRanks, QuotientGroupTable,
 QuotientKernels, QuotientKernelSpacings, QuotientPermutationGroups,
 QuotientPGroupGenerators, QuotientPGroupRanks, QuotientPGroupReduction,
 QuotientPGroupRedundancies, QuotientPGroupSizes, QuotientPGroupTables}
```

```
N2 (*antalet kvotgrupper i Z6*)
```

```
4
```

```
<< AdditiveGroupTheorems`
```

```
MakeGroup[12, True]
```

```
Estimated time is:           -6
9.46997 10 seconds.
Computation time: 0.016 seconds.
```

```
Containment[4]
```

```
{{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}, {0, 2, 4, 6, 8, 10}, {0, 4, 8}}
```

```
CorrespondenceMap[4]
```

```
⟨| {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11} → {0, 1, 2, 3},
 {0, 2, 4, 6, 8, 10} → {0, 2}, {0, 4, 8} → {0} |⟩
```

```
cm = UniqueCorrespondenceMapIndexed[4]
```

```
⟨| 1 → {{1, 4}}, 2 → {{2, 3}}, 4 → {{4, 1}} |⟩
```

```
cmi = Flatten /@ Values[cm]
```

```
{{1, 4}, {2, 3}, {4, 1}}
```

```
{QuotientGroups[][[1, 4]], QuotientGroups[][[2, 3]], QuotientGroups[][[4, 1]]}
```

```
{{0, 1, 2, 3}, {0, 2}, {0}}
```

```
(*en av två metoder för direktberäkning av korrespondens*)
CorrespondenceMethodGenerators[4]
```

```
⟨| 1 → {0, 1, 2, 3}, 2 → {0, 2}, 4 → {0} |⟩
```

```
ExtensionIsomorphy[][[3]]
(*vänster och högerled i andra isomorfisatsen för tredje undergruppen*)
```

```
{{0}, {0, 1}, {0}, {0, 1, 2, 3}, {0, 1}, {0, 1, 2, 3}}
```

```
ExtensionRelabeling[][[3]]
(*hur elementen ska döpas om för att andra isomorfisatsen ska synliggöras*)
```

```
{⟨| |⟩, ⟨| 0 → 0, 3 → 1 |⟩, ⟨| |⟩, ⟨| 0 → 0, 3 → 1, 6 → 2, 9 → 3 |⟩, ⟨| |⟩, ⟨| |⟩}
```

```
ProductsToIntersections[SubgroupProducts[]] // Short
```

```
{{{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11},
   {0, 2, 4, 6, 8, 10}, {0, 3, 6, 9}, {0, 4, 8}, {0, 6}, {0}},
  {{0, 2, 4, 6, 8, 10}, {0, 2, 4, 6, 8, 10}, {0, 6}, {0, 4, 8}, {0, 6}, {0}}, {<<1>>},
  {<<1>>}, {{0, 6}, {0, 6}, {0, 6}, {0}, {0, 6}, {0}}, {{0}, {0}, {0}, {0}, {0}, {0}}}
```

```
SubgroupIntersections[] // Short (*produkt och snitt är utbytbara*)
```

```
{{{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11},
   {0, 2, 4, 6, 8, 10}, {0, 3, 6, 9}, {0, 4, 8}, {0, 6}, {0}},
  {{0, 2, 4, 6, 8, 10}, {0, 2, 4, 6, 8, 10}, {0, 6}, {0, 4, 8}, {0, 6}, {0}}, {<<1>>},
  {<<1>>}, {{0, 6}, {0, 6}, {0, 6}, {0}, {0, 6}, {0}}, {{0}, {0}, {0}, {0}, {0}, {0}}}
```

```
(*tre första undergrupperna i rad uppfyller inte antaganden*)
ThirdIsomorphyMappingIso[1, 2, 3]
Last[ThirdIsomorphyMappingIso[1, 2, 3]]
```

```
{{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11} → {0, 1, 2},
 {0, 2, 4, 6, 8, 10} → {0, 1, 2}, {0, 1} → {0}}
```

```
{0, 1} → {0}
```

```
(*hoppar över till fjärde så ..*)
ThirdIsomorphyMappingIso[1, 2, 4]
Last[ThirdIsomorphyMappingIso[1, 2, 4]]
```

```
{{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11} → {0, 1, 2, 3},
 {0, 2, 4, 6, 8, 10} → {0, 1}, {0, 1} → {0, 1}}
```

```
{0, 1} → {0, 1}
```

```
Names["AdditiveGroupTheorems`*"]
```

```
{AdditiveGroupTheoremsPackage, AllContainmentCorrespondenceOrders,
 AllContainmentCorrespondencies, AllContainmentsCorrespondenceOrders,
 AllQuotientCorrespondenceOrders, AllQuotientCorrespondencies, Collisions,
 ContainmentCorrespondenceGroups, ContainmentCorrespondenceOrder,
 CorrespondenceMap, CorrespondenceMapIndexed, CorrespondenceMaxReductions,
 CorrespondenceMethodDictionary, CorrespondenceMethodGenerators,
 CorrespondenceReduction, CorrespondenceSizeFactors,
 ExtensionIsomorphy, ExtensionRelabeling, ExtensionRelabelingIndexed,
 ExtensionTransformativeTerms, FirstFalseKMindexed, FirstKMindexed, FirstKMs,
 FirstTrueKMindexed, IntersectionsToProducts, IsomorphicCyclicGroup,
 IsomorphicGroup, LargestThirdIsomorphyPair, ProductsToIntersections,
 QuotientCorrespondenceGroups, QuotientCorrespondenceOrder,
 QuotientSimilars, RelabelElements, SecondIsomorphyDeterminingSubgroups,
 ThirdIsomorphies, ThirdIsomorphyCheck, ThirdIsomorphyFromGenerators,
 ThirdIsomorphyMappingIndexToIso, ThirdIsomorphyMappingIso,
 ThirdIsomorphyTrippels, ThirdIsomorphyTrippelsClasses, ThirdIsomorphyTrippelsIso,
 ThirdNonIsomorphicK, ThirdNonIsomorphicKIndex, ThirdNonIsomorphicM,
 ThirdNonIsomorphicMIndex, ThirdNonIsomorphies, ThirdNonIsomorphiesFull,
 ThirdNonIsomorphiesN, UniqueCorrespondenceMap, UniqueCorrespondenceMapIndexed}
```